18.03SC Practice Problems 7

Sinusoids

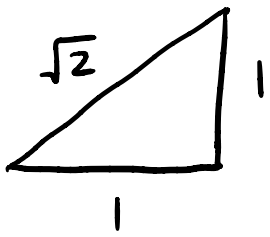
1. Write each of the following functions (of t) in the form $A\cos(\omega t - \phi)$. In each case, begin by drawing a right triangle with sides a and b.

(a) $\cos(2t) + \sin(2t)$.

(b) $\cos(\pi t) - \sqrt{3}\sin(\pi t)$.

(c) $\operatorname{Re}\dfrac{e^{it}}{2+2i}$.

(a)



$$\cos(\theta)+\sin(\theta) = \operatorname{Re}\{(a-bi)(\cos\theta+i\sin\theta)\}$$

$$= \operatorname{Re}\{\sqrt{a^2+b^2}\,e^{-i\phi}\cdot e^{i\theta}\}$$

$$= \operatorname{Re}\{\sqrt{a^2+b^2}\,e^{(\theta-\phi)i}\}$$

$$= \sqrt{a^2+b^2}\cos(\theta-\phi), \quad \theta = \omega t, \quad \phi = \tan^{-1}\frac{b}{a}$$

$$\cos(2t)+\sin(2t)$$

$$= \sqrt{2}\cos(2t-\tan^{-1}1)$$

$$= \sqrt{2}\cos\left(2t-\frac{\pi}{4}\right)$$

(b)

$$\cos\pi t - \sqrt{3}\sin\pi t$$

$$= 2\cos\left(\pi t-\left(-\frac{\pi}{3}\right)\right)$$

$$= 2\cos\left(\pi t+\frac{\pi}{3}\right)$$

(c)

$$\frac{e^{it}}{2+2i}$$

$$= \frac{e^{it}}{2\sqrt{2}\,e^{i\frac{\pi}{4}}}$$

$$= \frac{\sqrt{2}}{4}e^{(t-\frac{\pi}{4})i}$$

$$= \frac{\sqrt{2}}{4}\cos\left(t-\frac{\pi}{4}\right)$$